



Enhancing Reuse of Constraint Solutions to Improve Symbolic Execution

Xiangyang Jia (Wuhan University)
Carlo Ghezzi (Politecnico di Milano)
Shi Ying (Wuhan University)

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- ❖ **Motivation**
- ❖ **Logical Basis of our Approach**
- ❖ **GreenTrie Framework**
 - ❖ **Constraint Reduction**
 - ❖ **Constraint Storing**
 - ❖ **Constraint Querying**
- ❖ **Evaluation**
- ❖ **Conclusion and Future Work**

Motivation

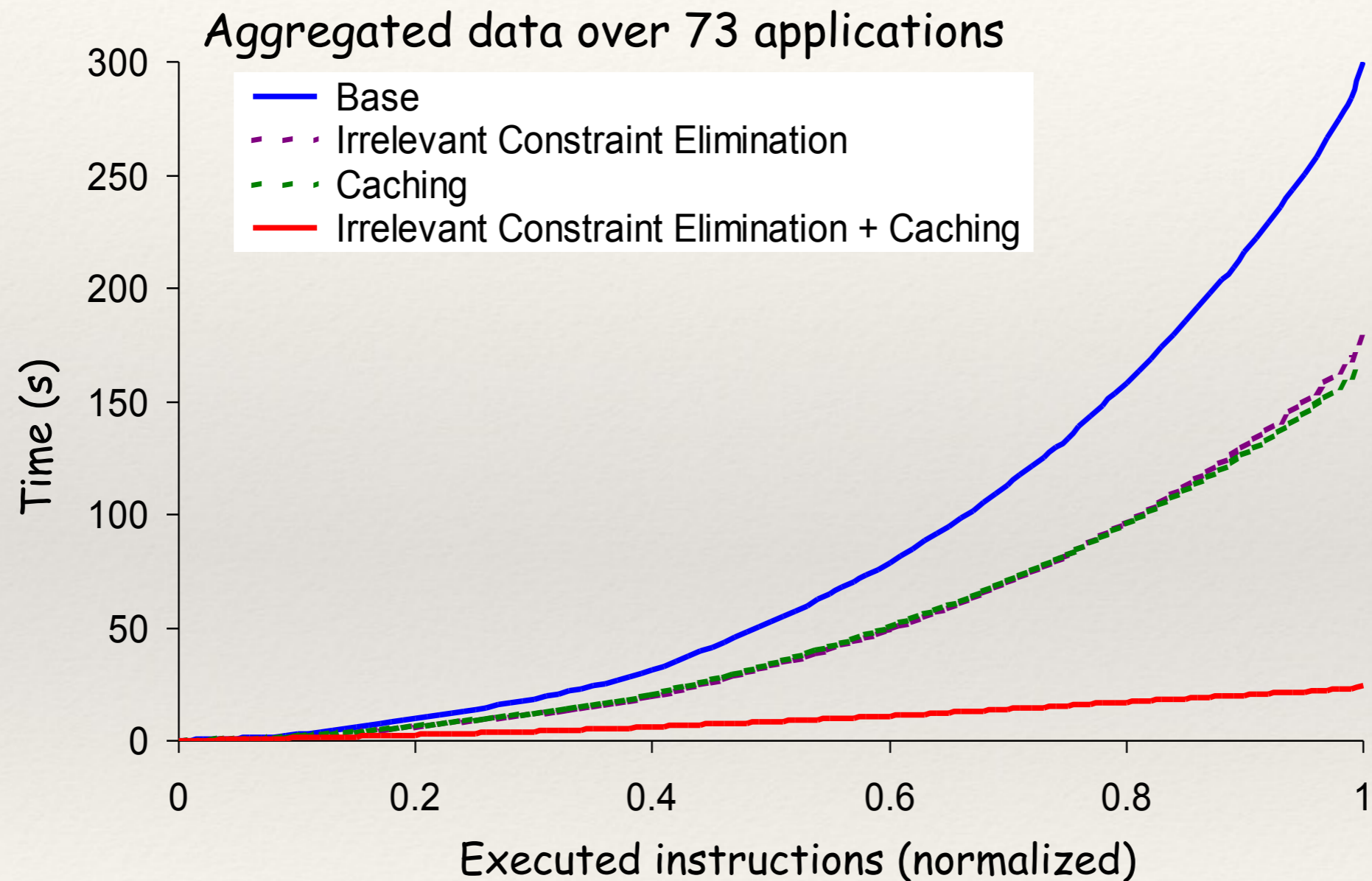
- ❖ **Symbolic Execution(SE)**

- ❖ A well-known program analysis technique, mainly used for test-case generation and bug finding.

- ❖ **Constraint Solving**

- ❖ The most time-consuming work in SE
 - ❖ Optimization approaches:
 - ❖ Irrelevant constraint elimination
 - ❖ Caching and reuse

Motivation



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[From Shauvik Roy Choudhary's Slides]

Motivation

❖ Reuse of Constraint Solutions

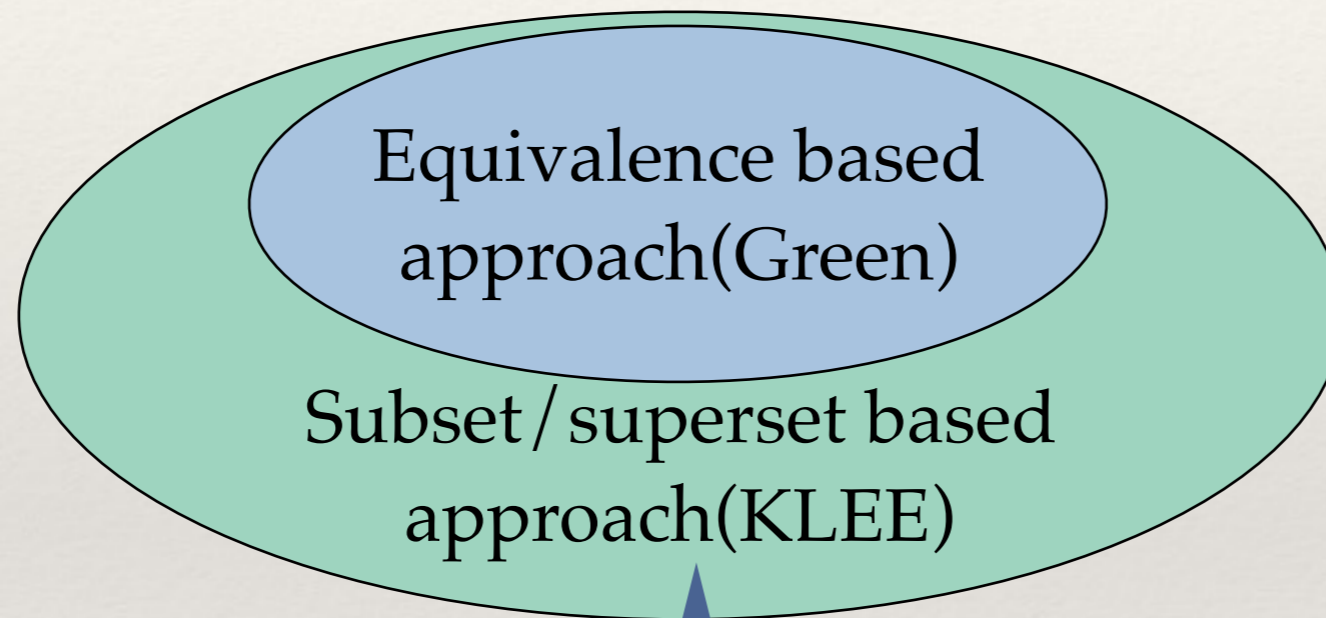
Equivalence based
approach(Green)

$x > 0$ is equivalent to $y > 0$

$x + 1 > 0 \wedge x \leq 1$ is equivalent to $y < 2 \wedge y \geq 0$ (if x, y are integers)

Motivation

❖ Reuse of Constraint Solutions

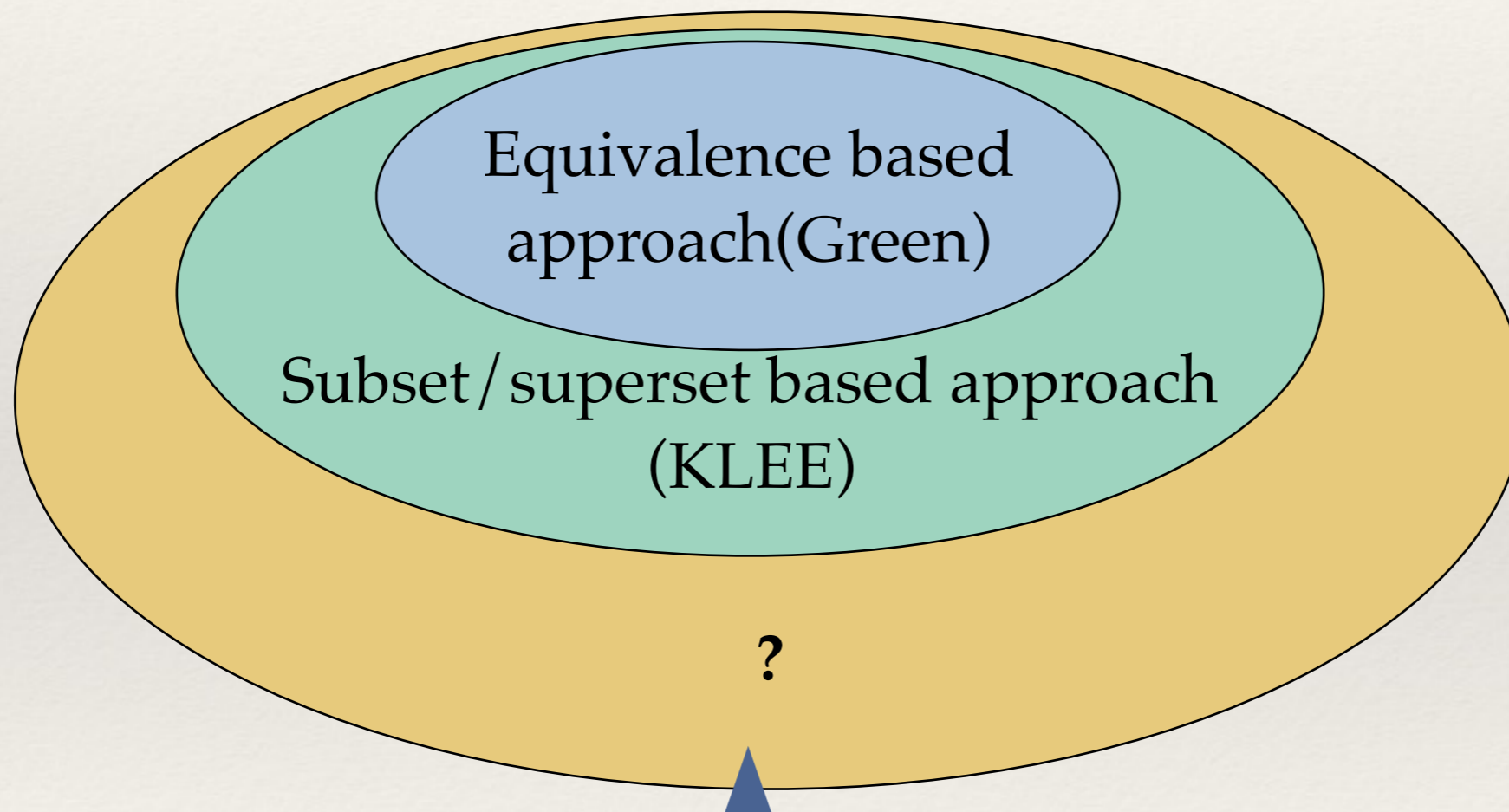


If $A \wedge B \wedge C$ is satisfiable, then $A \wedge B$ is satisfiable

If $A \wedge B \wedge C$ is unsatisfiable, then $A \wedge B \wedge C \wedge D$ is unsatisfiable

Motivation

❖ Reuse of Constraint Solutions

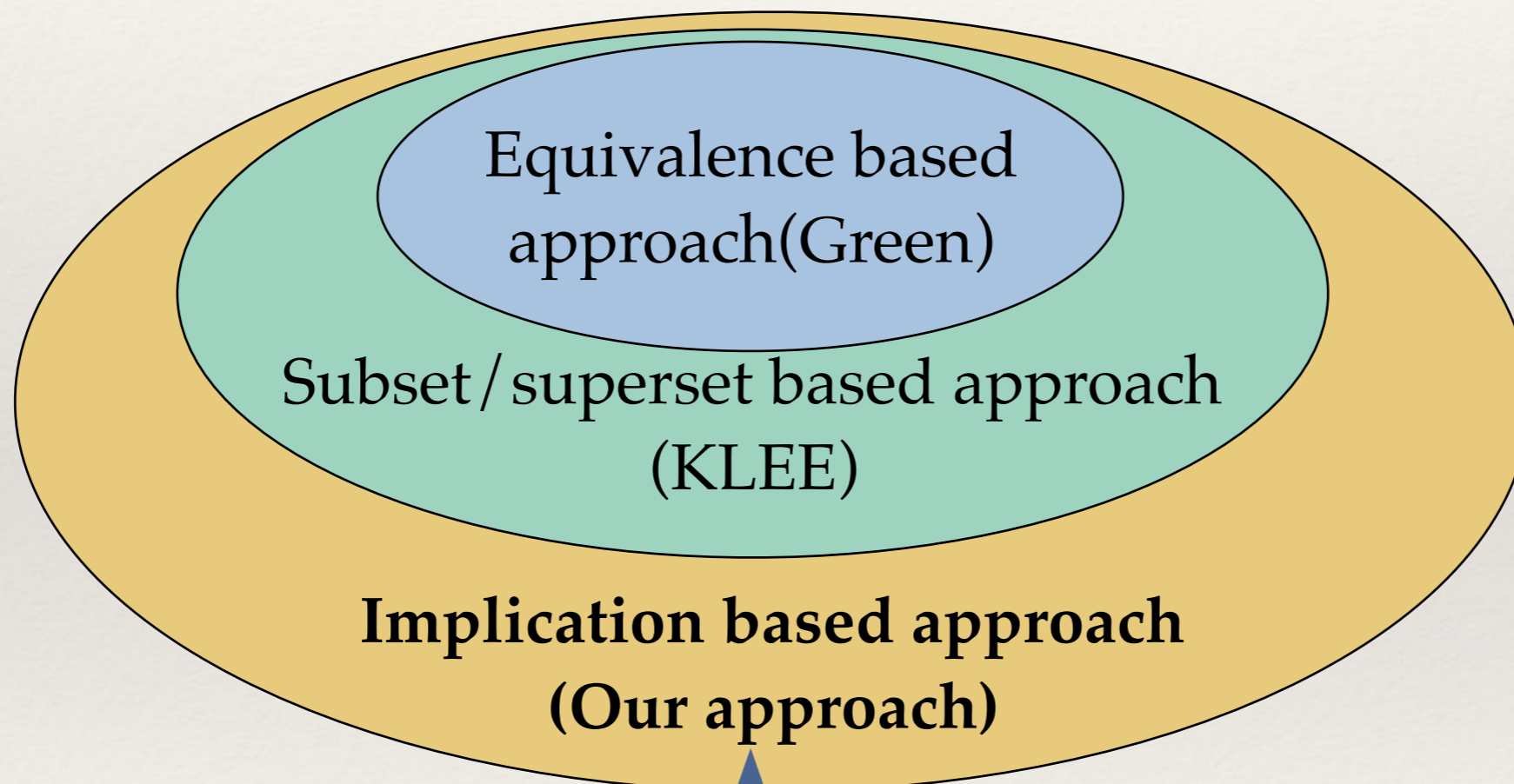


If $x > 0$ is satisfiable, can we prove $x > -1$ satisfiable?

If $x < 0 \wedge x > 1$ is unsatisfiable, can we prove $x < -1 \wedge x > 2$ unsatisfiable?

Motivation

❖ Reuse of Constraint Solutions



If $x > 0$ is satisfiable, can we prove $x > -1$ satisfiable?

If $x < 0 \wedge x > 1$ is unsatisfiable, can we prove $x < -1 \wedge x > 2$ unsatisfiable?

Logical Basis of our Approach

Implication and Satisfiability

Providing $C1 \rightarrow C2$

- if $C1$ is satisfiable, $C2$ is satisfiable
- if $C2$ is unsatisfiable, $C1$ is unsatisfiable

It looks easy to apply it to constraint reuse!

However, there is a problem:

Implication checking with SAT/SMT solver is even more expensive than only solving the single constraint itself.

Logical Basis of our Approach

- **The subset/superset (KLEE)**
 - $\{c1, c2\} \subseteq \{c1, c2, c3\}$ means $c1 \wedge c2 \wedge c3 \rightarrow c1 \wedge c2$
- **Logical subset/superset**
 - Given two constraint sets X, Y , if $\forall_{a \in X} \exists_{b \in Y} (b \rightarrow a)$, then X is a logical subset of Y , and Y is a logical superset of X
 - E.g: $X = \{m \neq 0, m > -1, m < 2\}$, $Y = \{m > 1, m < 2\}$
 - It is easy to prove that $(m > 1 \wedge m < 2) \rightarrow (m \neq 0 \wedge m > -1 \wedge m < 2)$

the subset/superset is a specific case of logical subset/superset

Logical subset/superset checks more implication cases!

- ❖ the two sets might have totally different atomic constraints
- ❖ the length of logical superset may be shorter than its subset

Logical Basis of our Approach

- Implication checking rules for atomic constraints**

$$(R1) \frac{}{C \rightarrow C} \qquad (R2) \frac{n \neq n'}{P + n = 0 \rightarrow P + n' \neq 0}$$

$$(R3) \frac{n \geq n'}{P + n = 0 \rightarrow P + n' \leq 0} \qquad (R4) \frac{n \leq n'}{P + n = 0 \rightarrow P + n' \geq 0}$$

$$(R5) \frac{n > n'}{P + n \leq 0 \rightarrow P + n' \neq 0} \qquad (R6) \frac{n > n'}{P + n \leq 0 \rightarrow P + n' \leq 0}$$

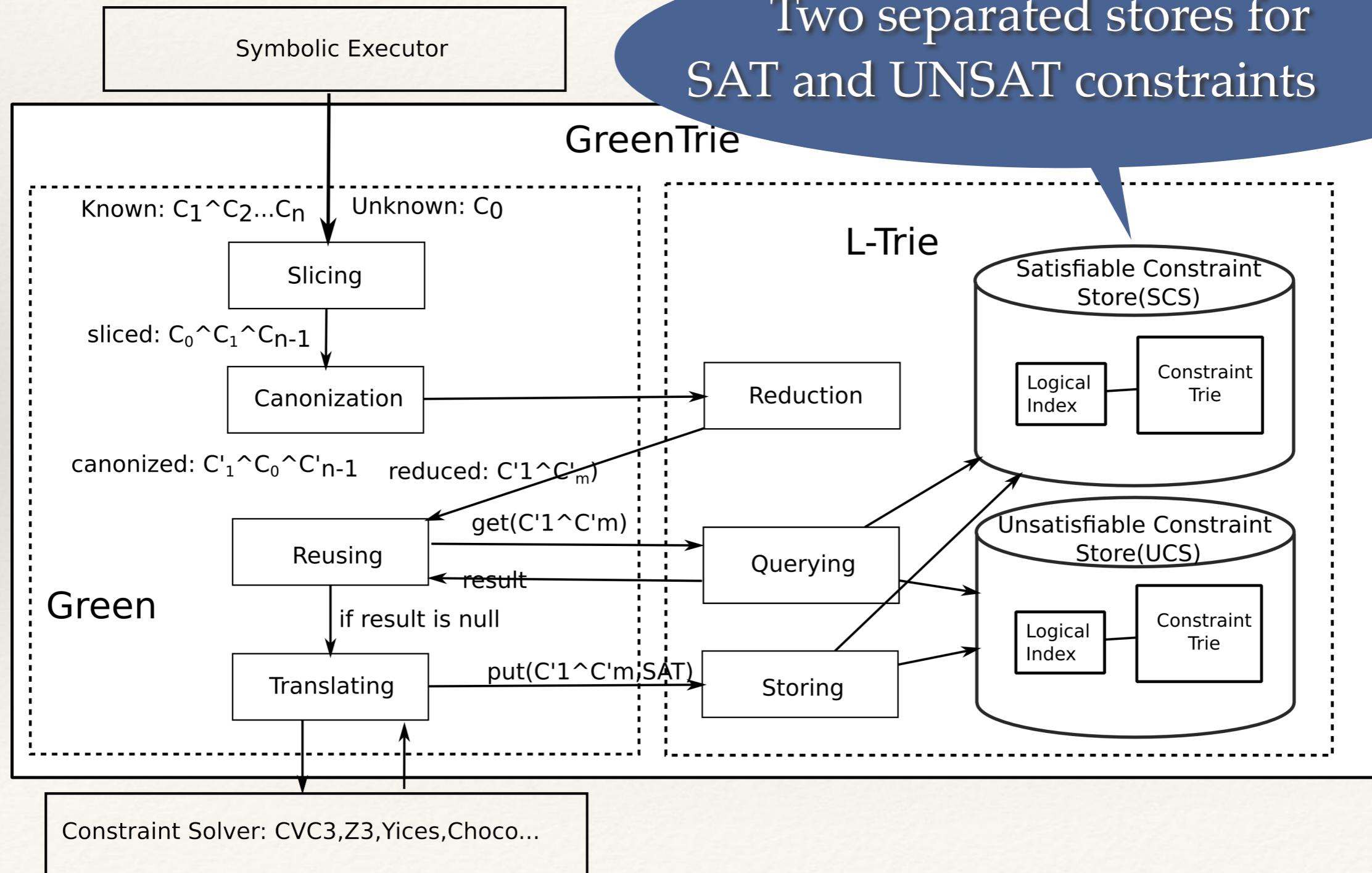
$$(R7) \frac{n < n'}{P + n \geq 0 \rightarrow P + n' \neq 0} \qquad (R8) \frac{n < n'}{P + n \geq 0 \rightarrow P + n' \geq 0}$$

P: non-constant prefix, n: constant number

E.g. $x+y+3 \geq 0$ has a non-constant prefix $x+y$ and a constant number 3

GreenTrie Framework

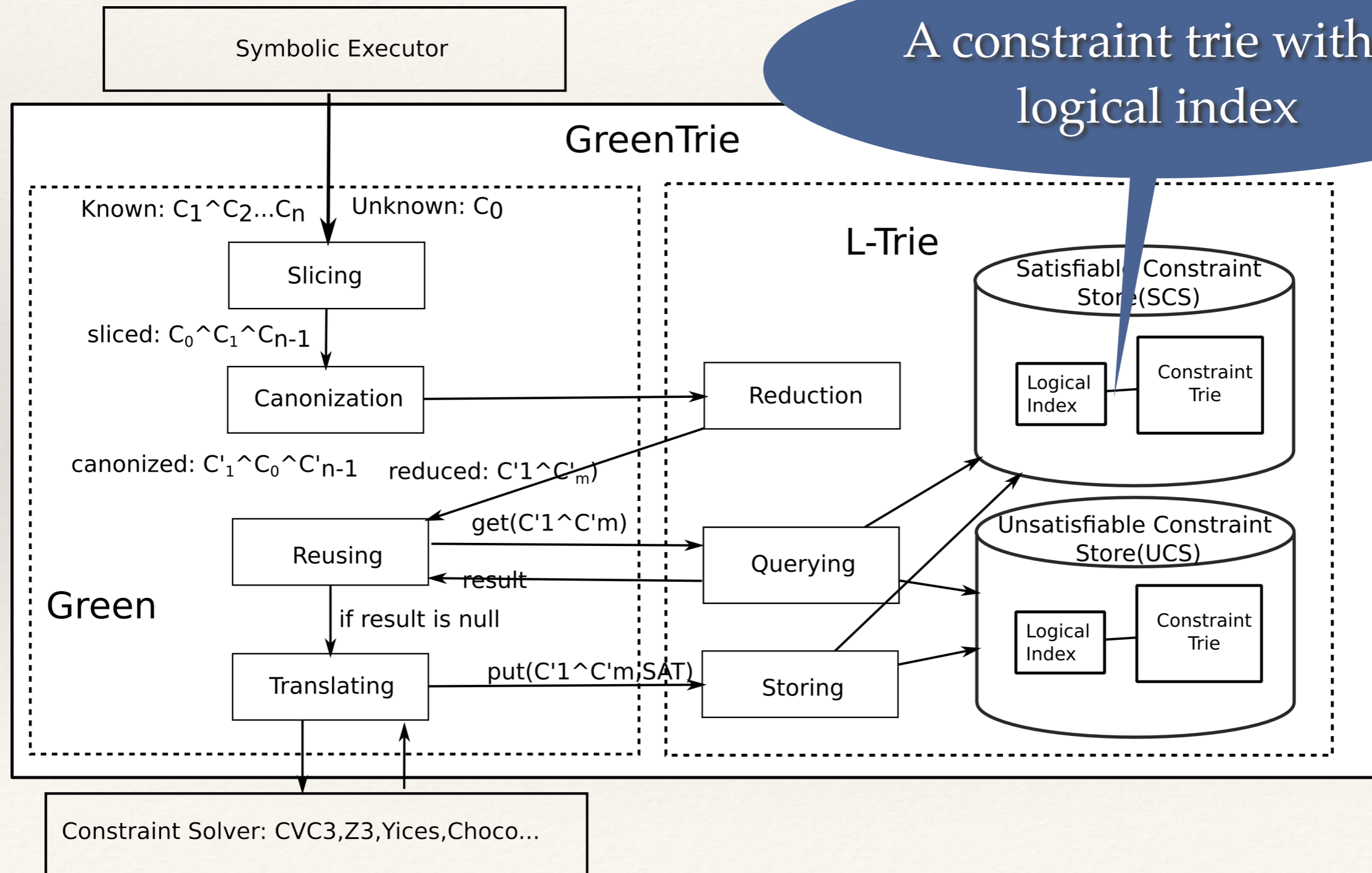
- Architecture of GreenTrie



Two separated stores for SAT and UNSAT constraints

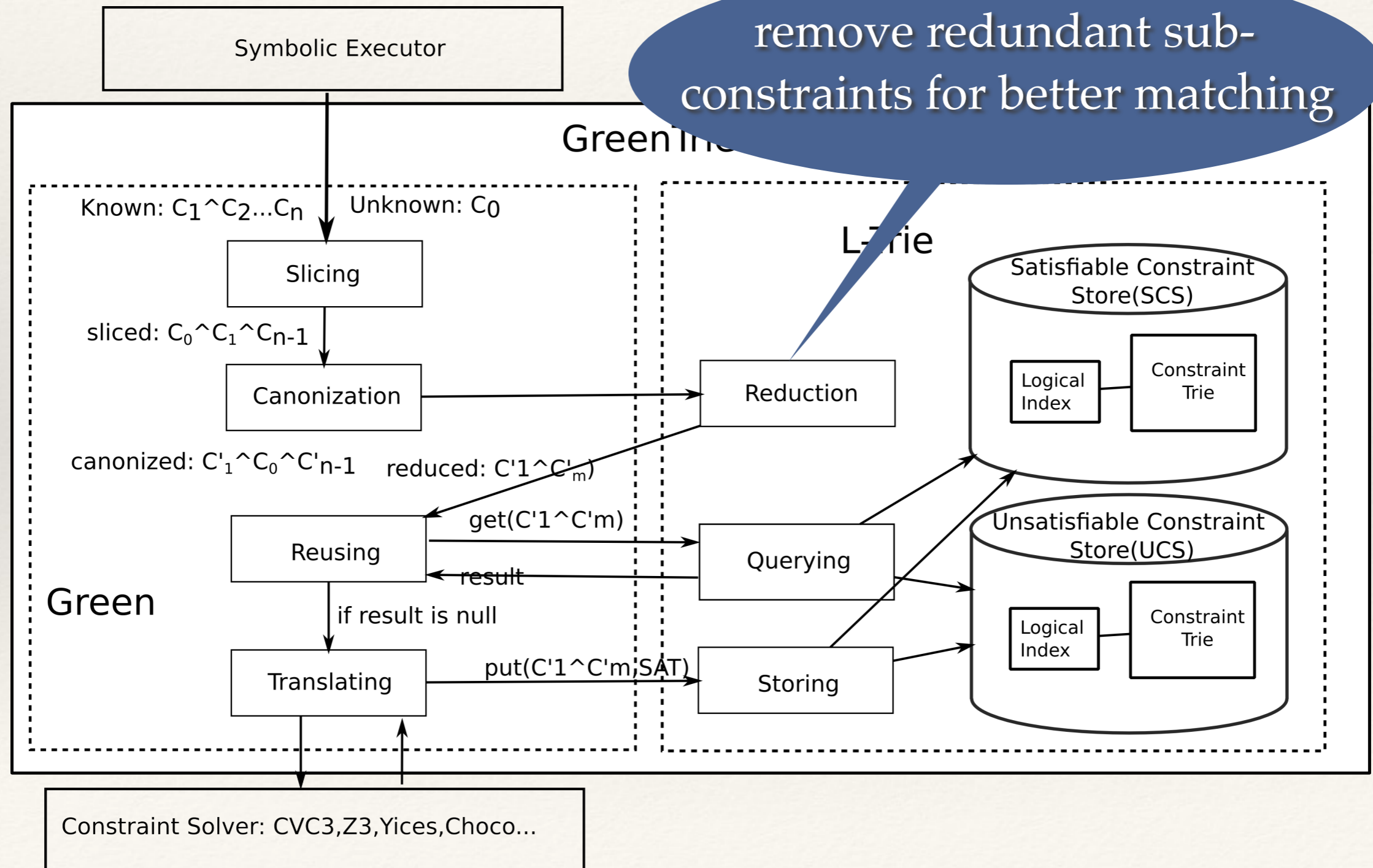
GreenTrie Framework

- Architecture of GreenTrie



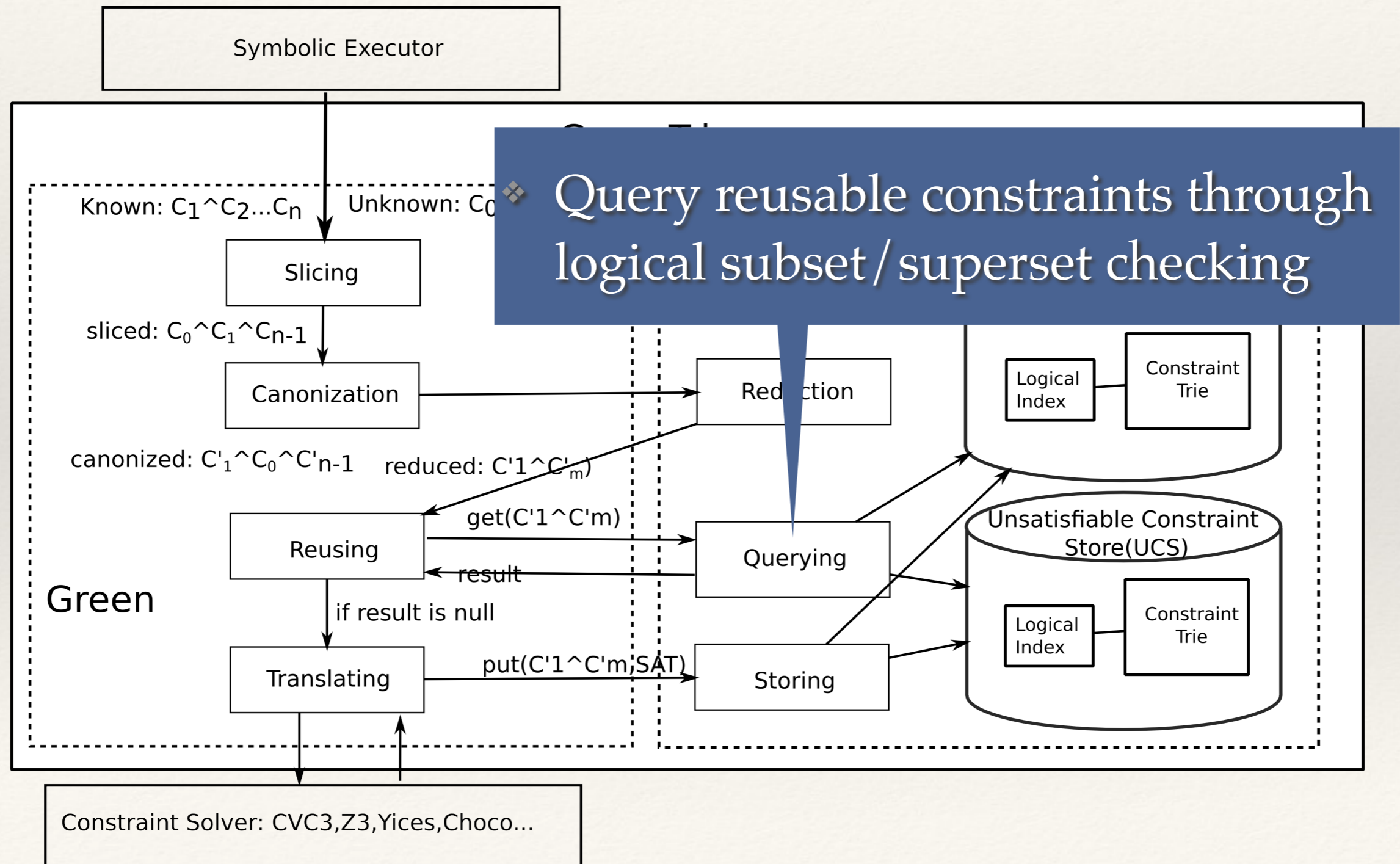
GreenTrie Framework

- Architecture of GreenTrie



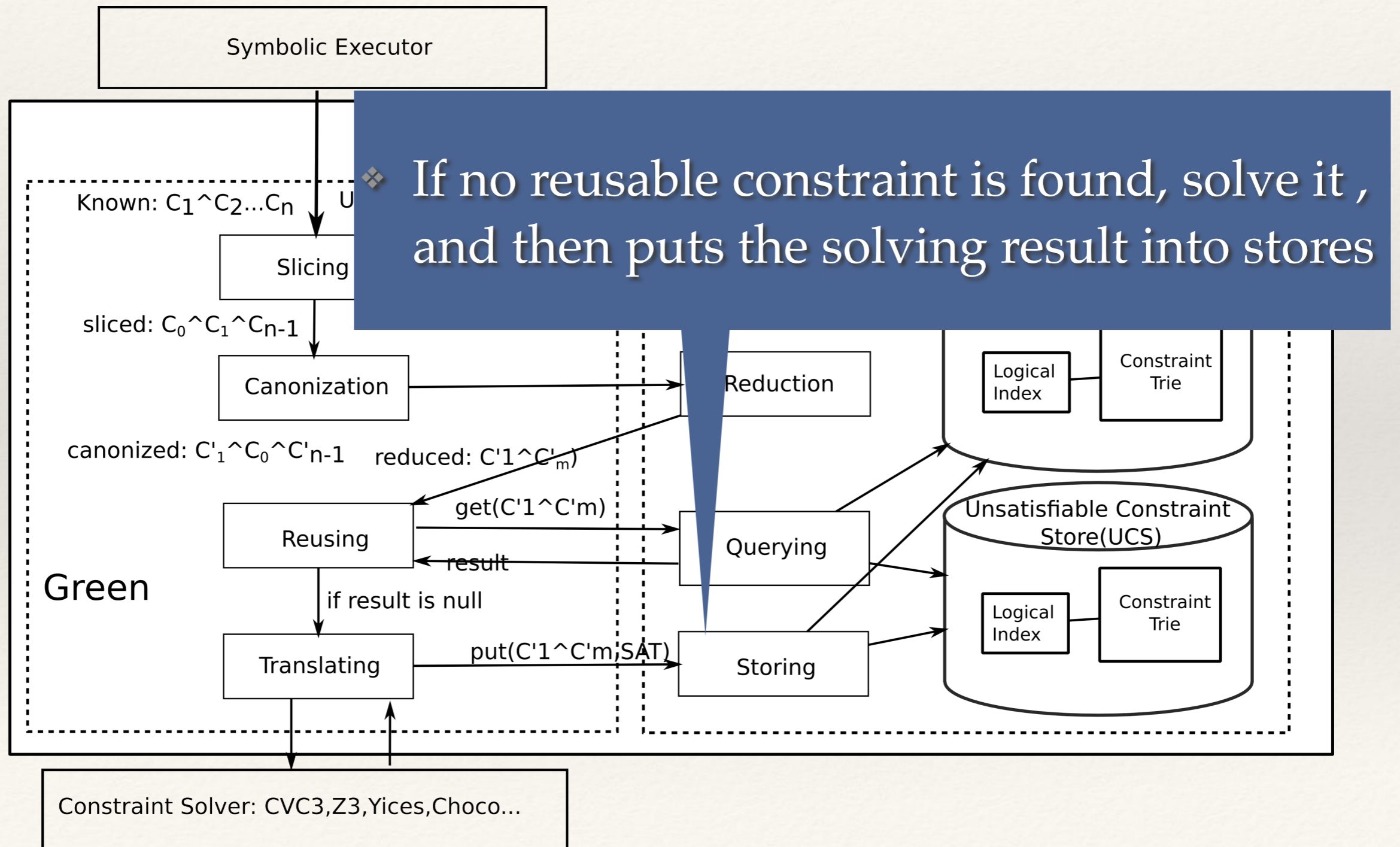
GreenTrie Framework

- Architecture of GreenTrie



GreenTrie Framework

- Architecture of GreenTrie



Constraint Reduction

- **Constraint Reduction**
 - **target: remove redundant sub-constraints**
 - **idea: interval computation-based constraint reduction**

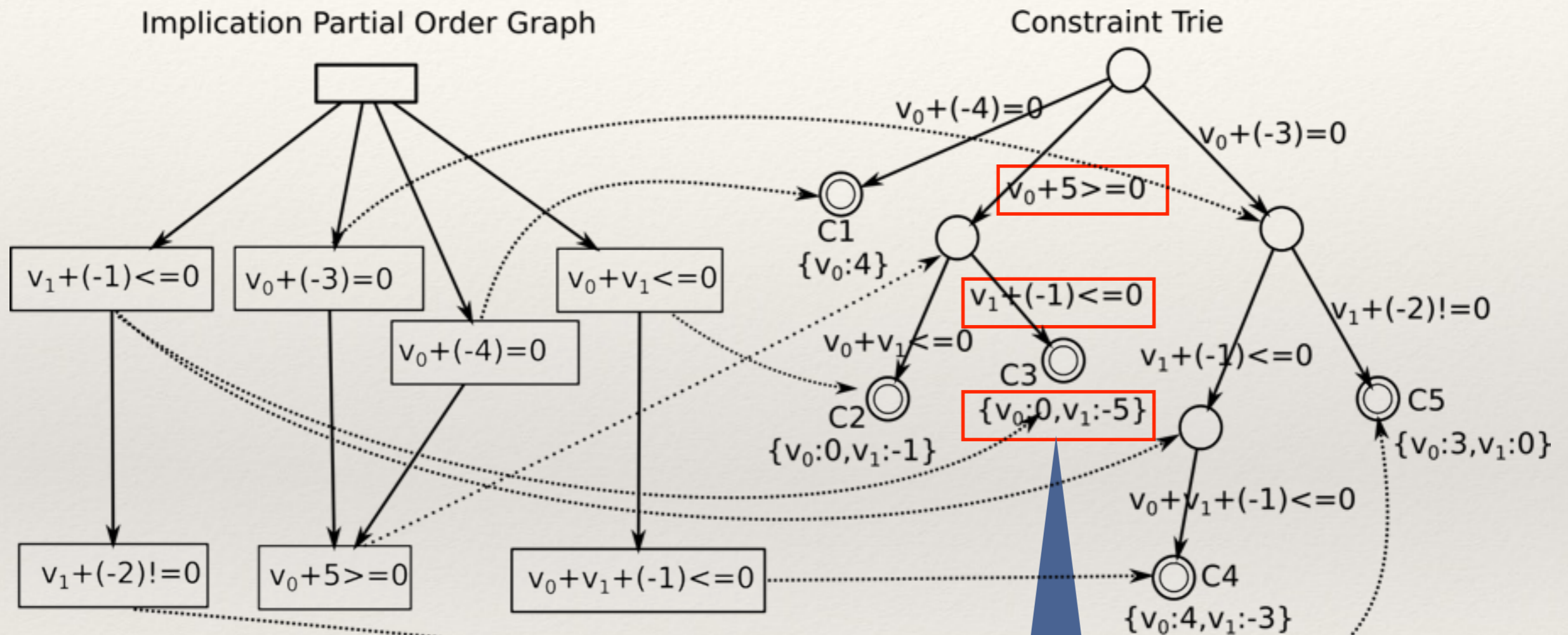
Example

$x+y+3 \geq 0 \wedge x+y+5 \geq 0 \wedge x+y-4 \leq 0 \wedge x+y \neq 0 \wedge x+y+6 \neq 0 \wedge x+y-4 \neq 0$

compute: $[-3, \infty) \cap [-5, \infty) \cap (-\infty, 4] - \{0, -6, 4\} = [-3, 4) - \{0\}$

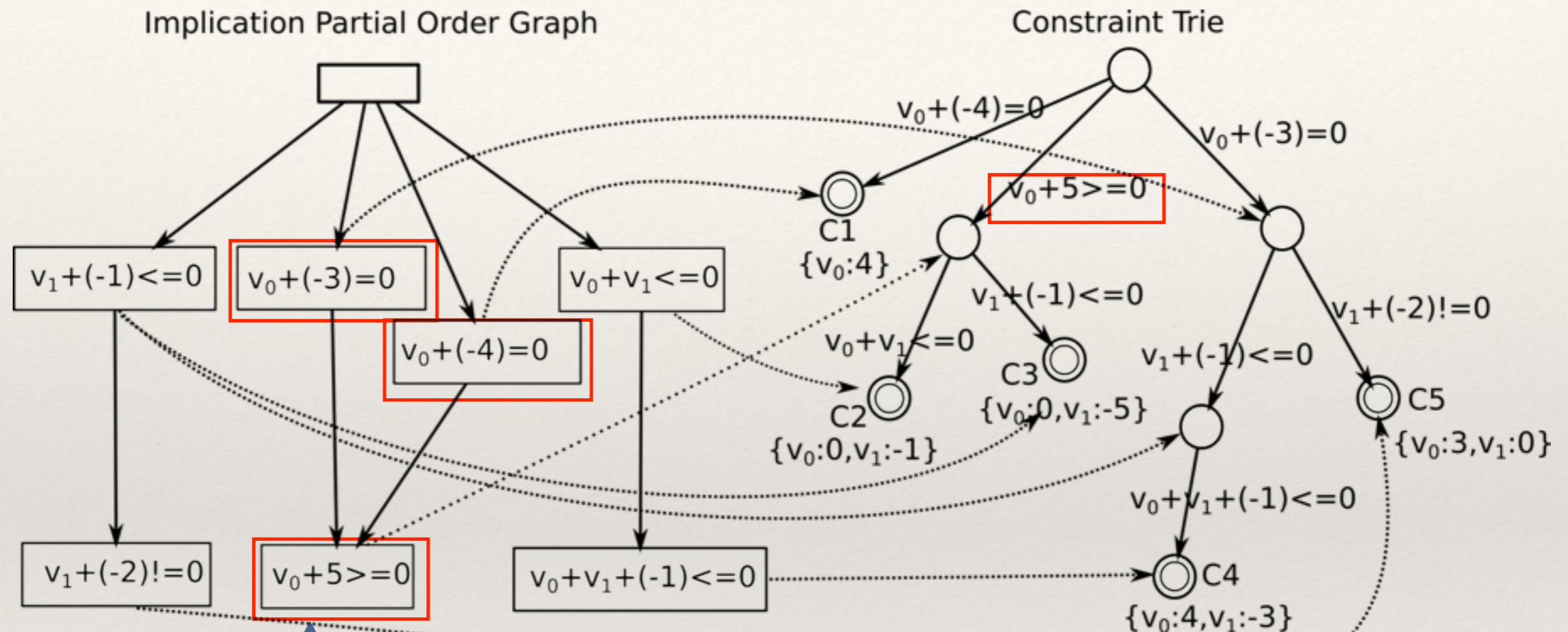
reduced: $x+y+3 \geq 0 \wedge x+y-4 < 0 \wedge x+y \neq 0$

Constraint Storing



- ❖ C3 represents a constraint $V_0 + 5 \geq 0 \wedge V_1 + (-1) \leq 0$, which has a solution $\{v_0:0, v_1:-5\}$

Constraint Storing

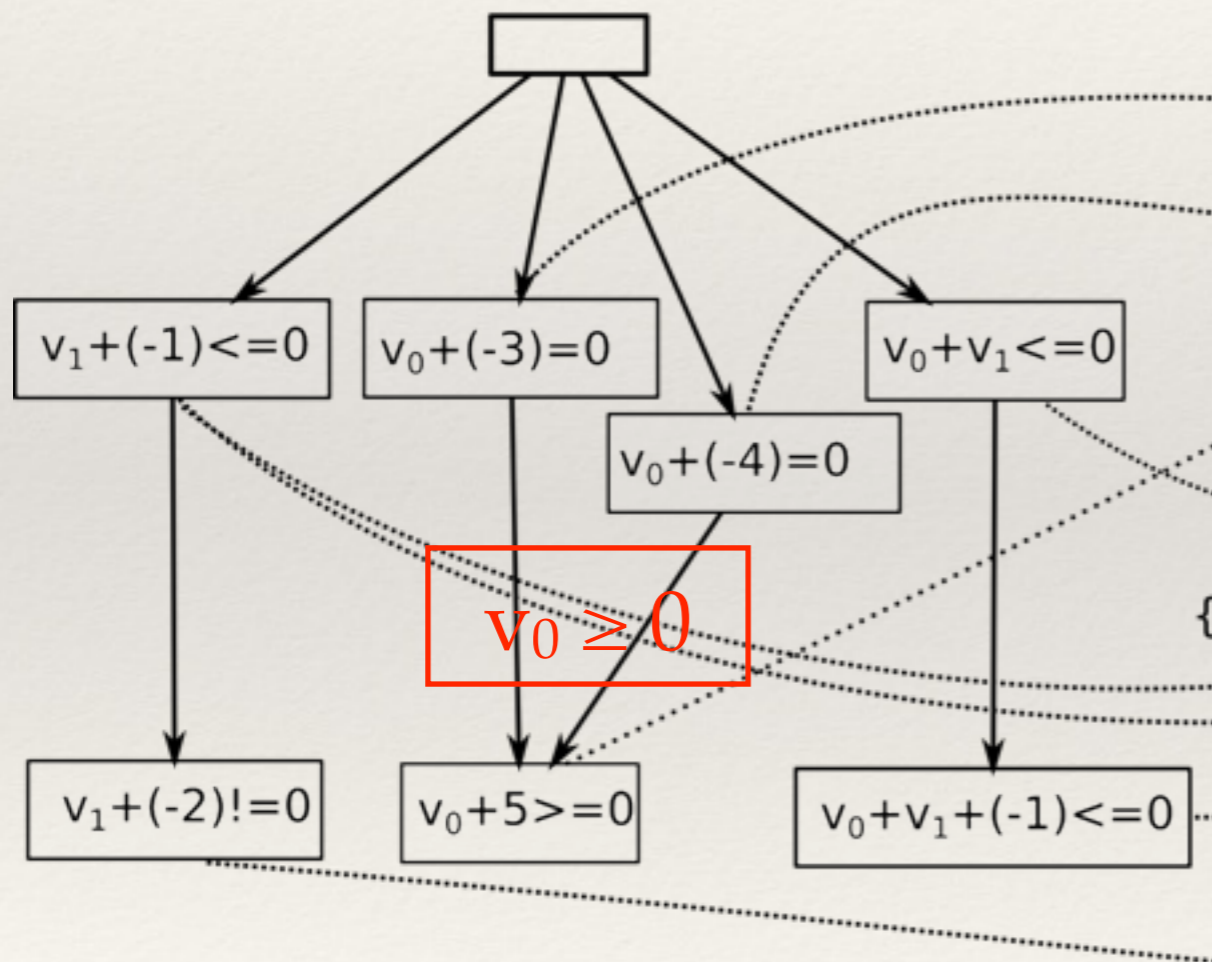


- ❖ $v_0 + 5 \geq 0$ is implied by $v_0 + (-3) = 0$ and $v_0 + (-4) = 0$
- ❖ $v_0 + 5 \geq 0$ has one occurrence in the trie, therefore it has a reference to the successive trie node.

Constraint Querying

❖ Implication Set(IS) and Reverse Implication Set(RIS)

Implication Partial Order Graph



Example

Constraint: $v_0 \geq 0$

$IS_{v_0 \geq 0}: \{v_0 + 5 \geq 0\}$

$RIS_{v_0 \geq 0}: \{v_0 + (-3) = 0, v_0 + (-4) = 0\}$

Constraint Querying

❖ Logical Superset Checking Algorithm

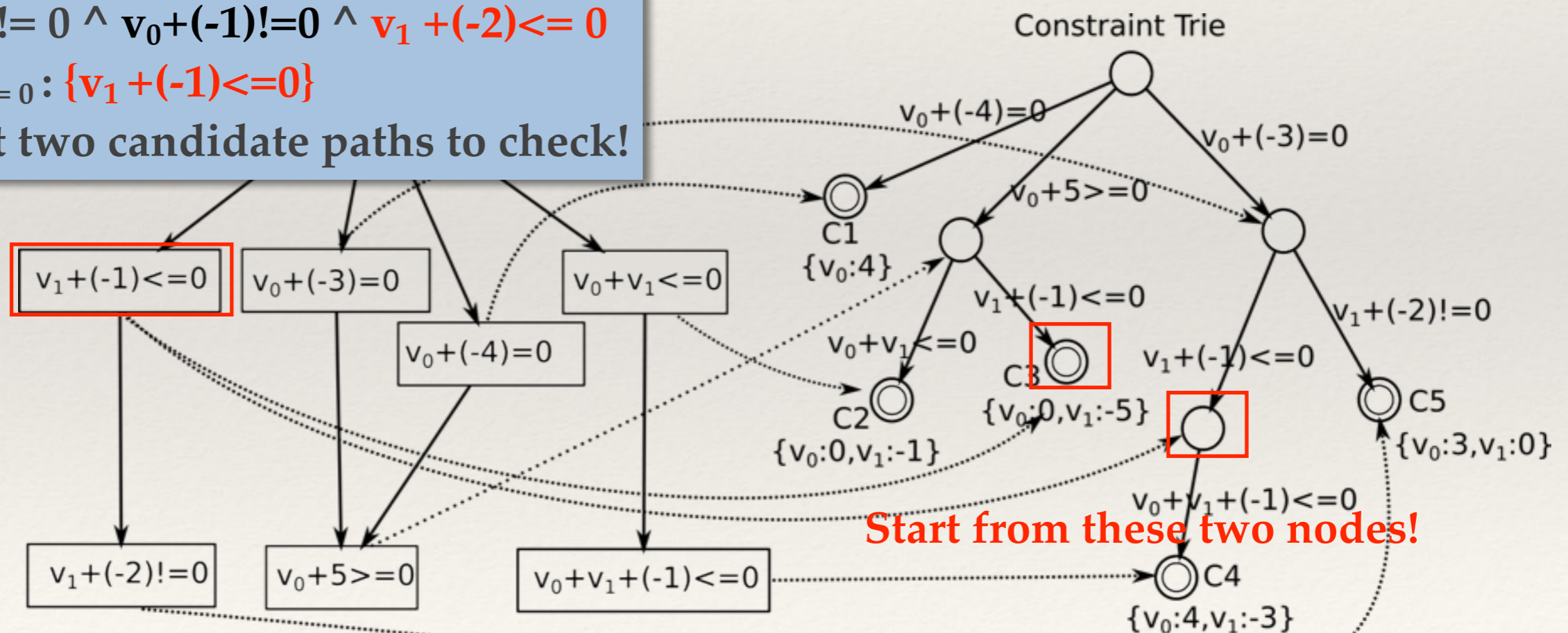
- ❖ Find a path in trie, so that every sub-constraint in target constraint is implied by at least one constraint on this path

Example

Target: $v_0 \neq 0 \wedge v_0 + (-1) \neq 0 \wedge v_1 + (-2) \leq 0$

$RIS_{v_1 + (-2) \leq 0} : \{v_1 + (-1) \leq 0\}$

So, we got two candidate paths to check!



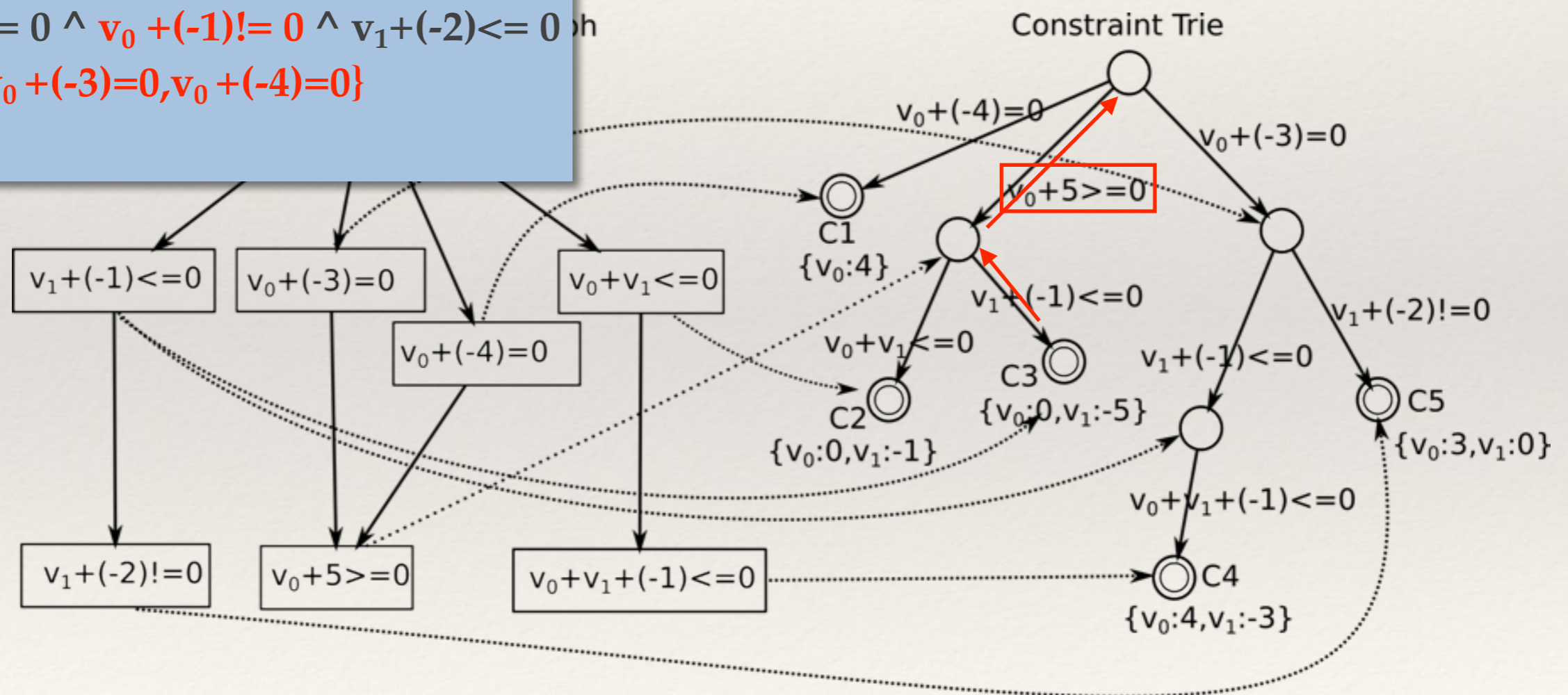
Constraint Querying

❖ Logical Superset Checking Algorithm

Example

Target : $v_0 \neq 0 \wedge v_0 + (-1) \neq 0 \wedge v_1 + (-2) \leq 0$

$RIS_{v_0 \neq 1} : \{v_0 + (-3) = 0, v_0 + (-4) = 0\}$



$v_0+5 \geq 0$ is not in the RIS,
the trie root is reached,
so this path doesn't match!

Constraint Querying

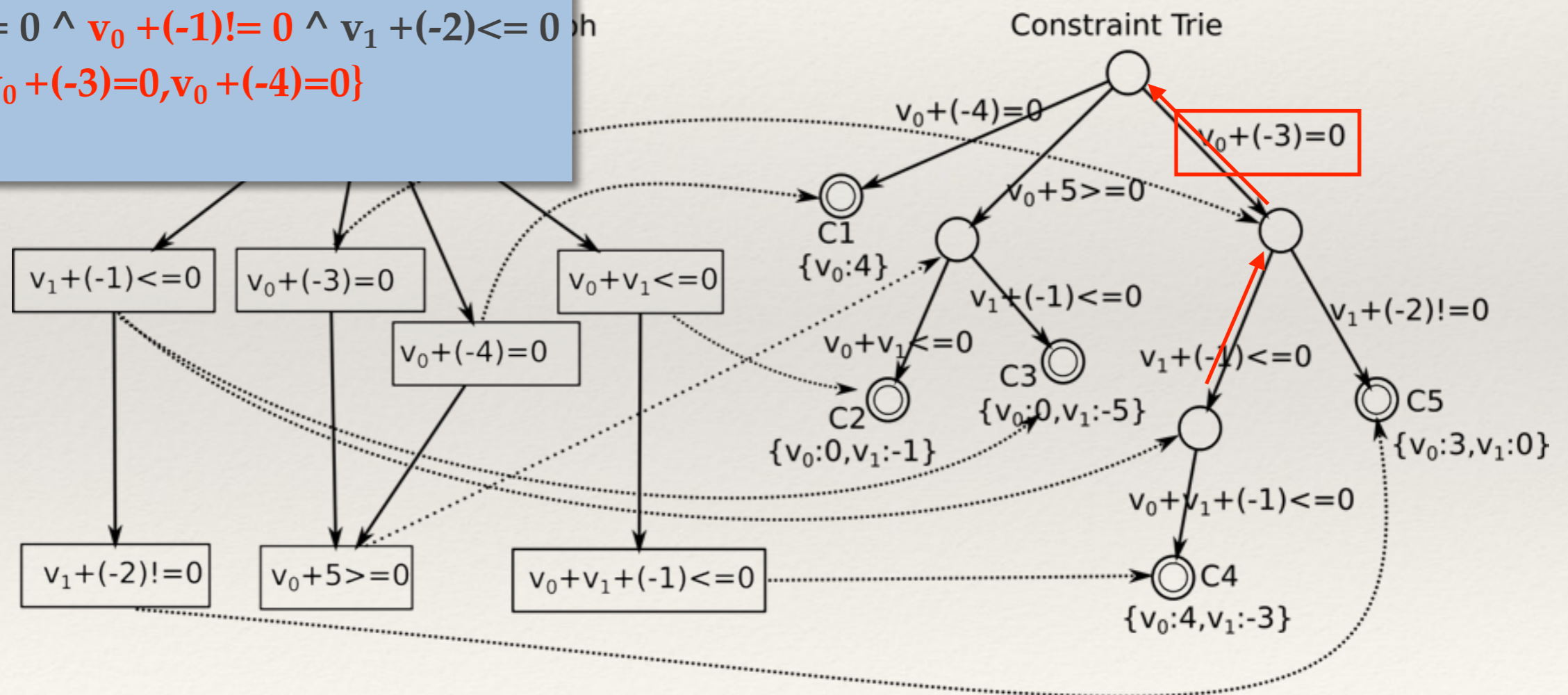
❖ Logical Superset Checking Algorithm

Example

Target: $v_0 \neq 0 \wedge v_0 + (-1) \neq 0 \wedge v_1 + (-2) \leq 0$

$RIS_{v_0 \neq 1} : \{v_0 + (-3) = 0, v_0 + (-4) = 0\}$

$v_0 + (-3) \geq 0$ is in the RIS,
go on to check next sub-constraint
of target!



Constraint Querying

❖ Logical Superset Checking Algorithm

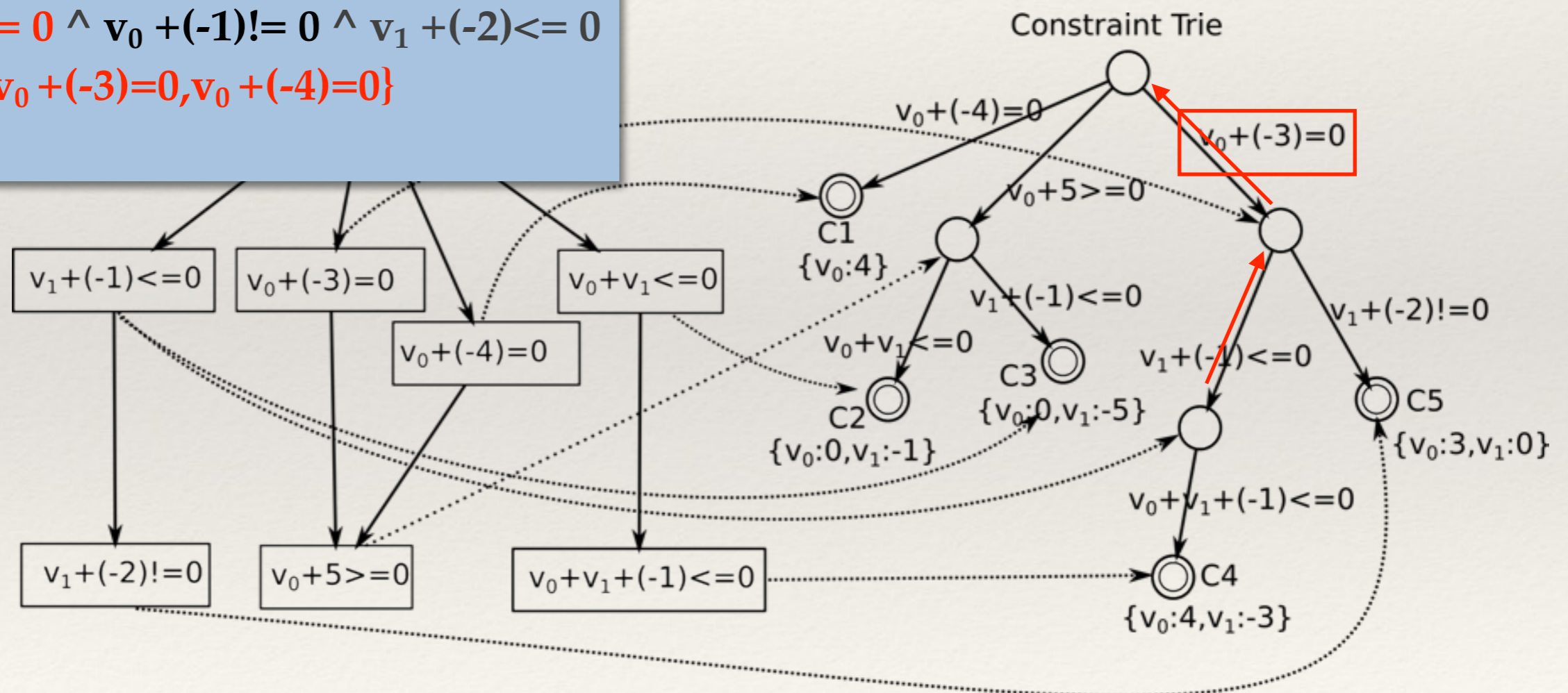
Example

Target: $v_0 \neq 0 \wedge v_0 + (-1) \neq 0 \wedge v_1 + (-2) \leq 0$

$RIS_{v_0 \neq 0} : \{v_0 + (-3) = 0, v_0 + (-4) = 0\}$

$v_0 + (-3) \geq 0$ is also in the RIS of $v_0 \neq 0$,
now, every sub-constraint in target is
implied by one constraint on this path.

C4 is the reusable constraint!



Constraint Querying

❖ Logical Subset Checking Algorithm

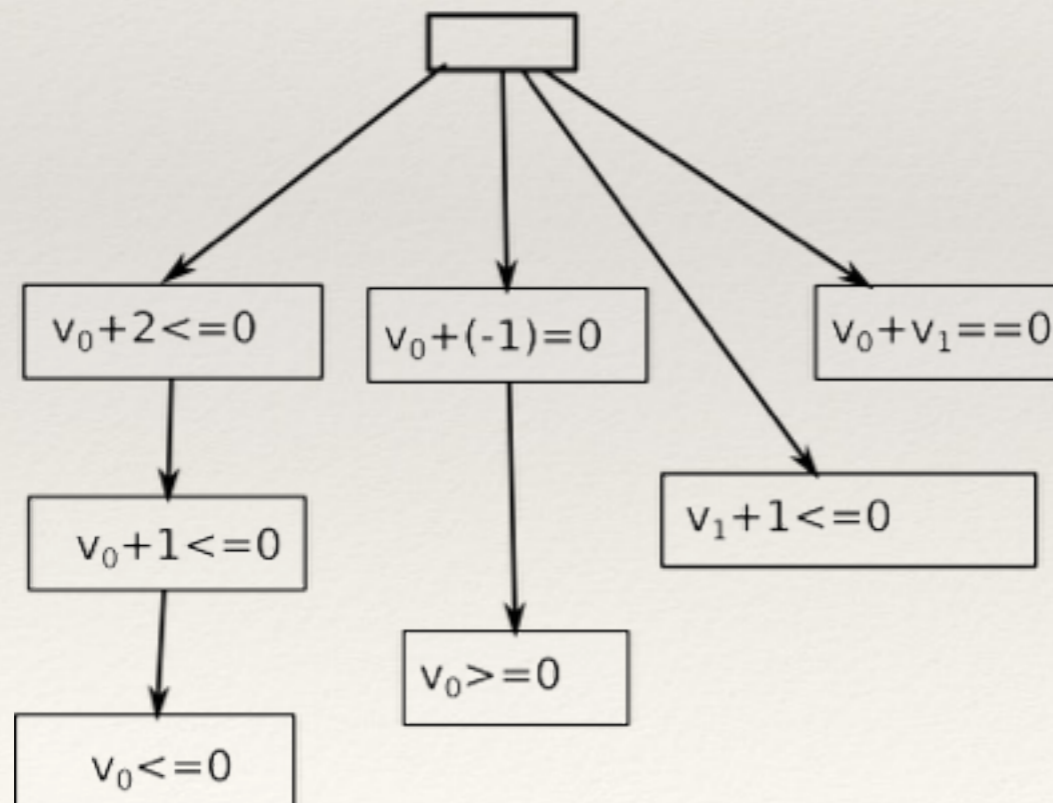
Target: $v_0 + (-1) \geq 0 \wedge v_0 + 3 \neq 0 \wedge v_0 + 4 \leq 0$

Union of ISs of the sub-constraints : $\{v_0 \geq 0\} \cup \{\} \cup \{v_0 + 2 \leq 0, v_0 + 1 \leq 0\}$

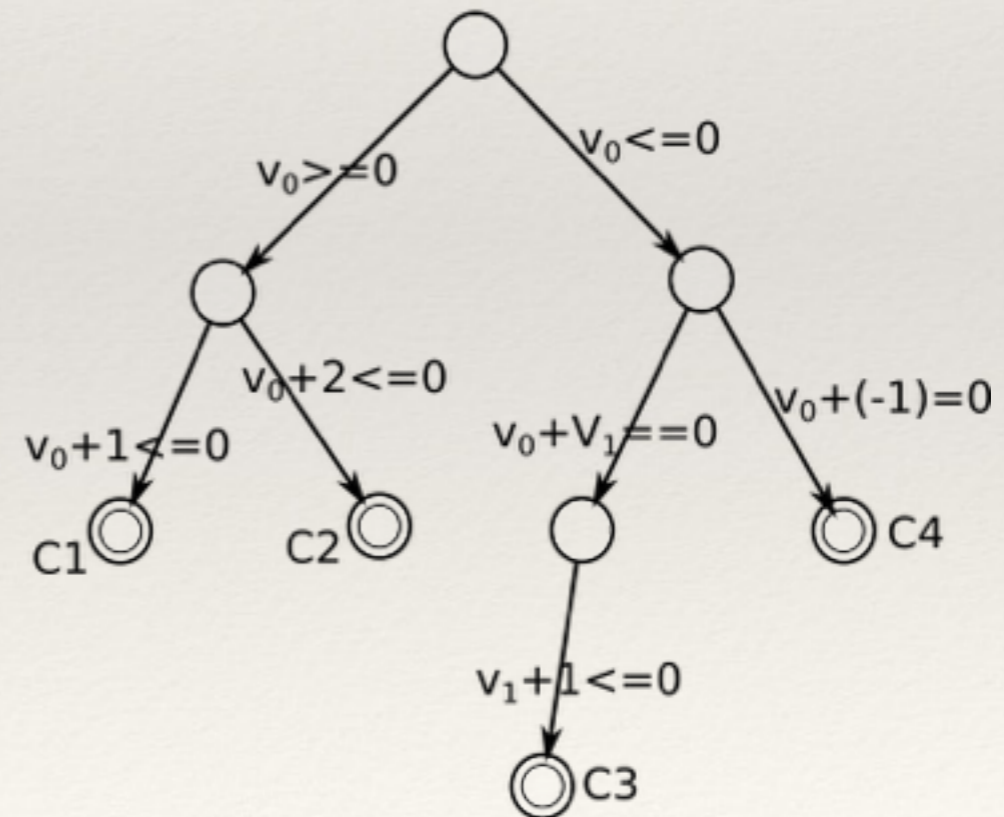
$IS_{\text{union}} = \{v_0 \geq 0, v_0 + 2 \leq 0, v_0 + 1 \leq 0\}$

We will find a trie path, so that all its sub-constraints on the path exists in IS_{union}

Implication Partial Order Graph



Constraint Trie



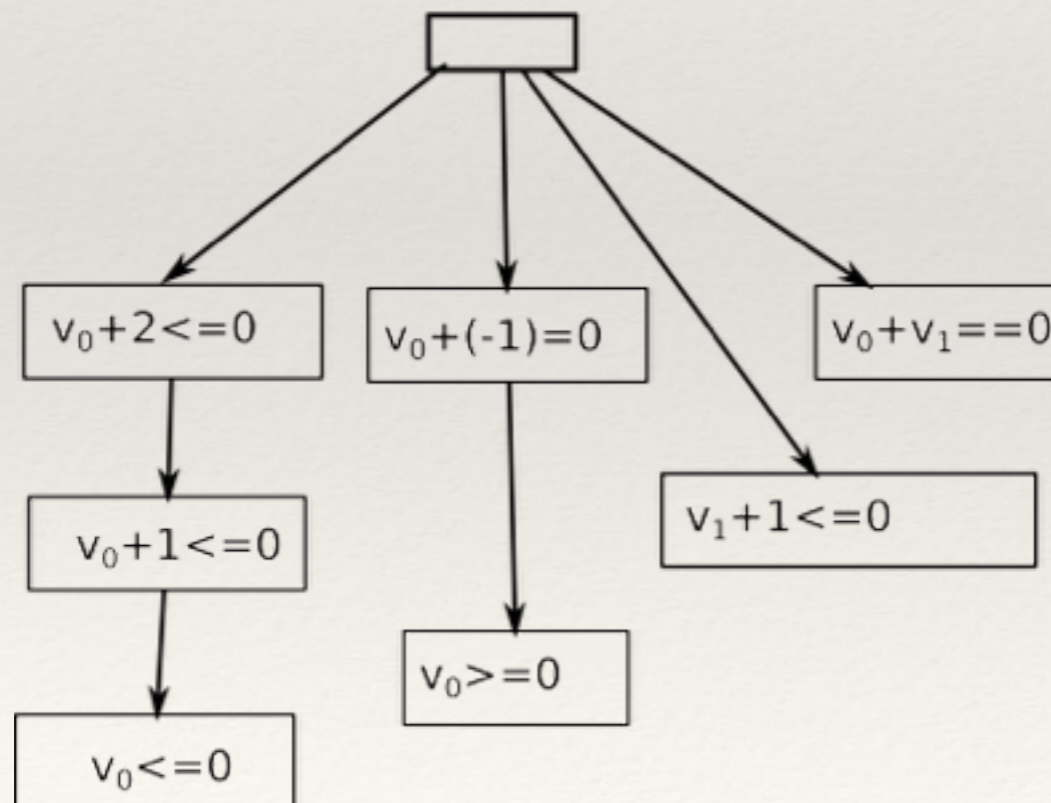
Constraint Querying

❖ Logical Subset Checking Algorithm

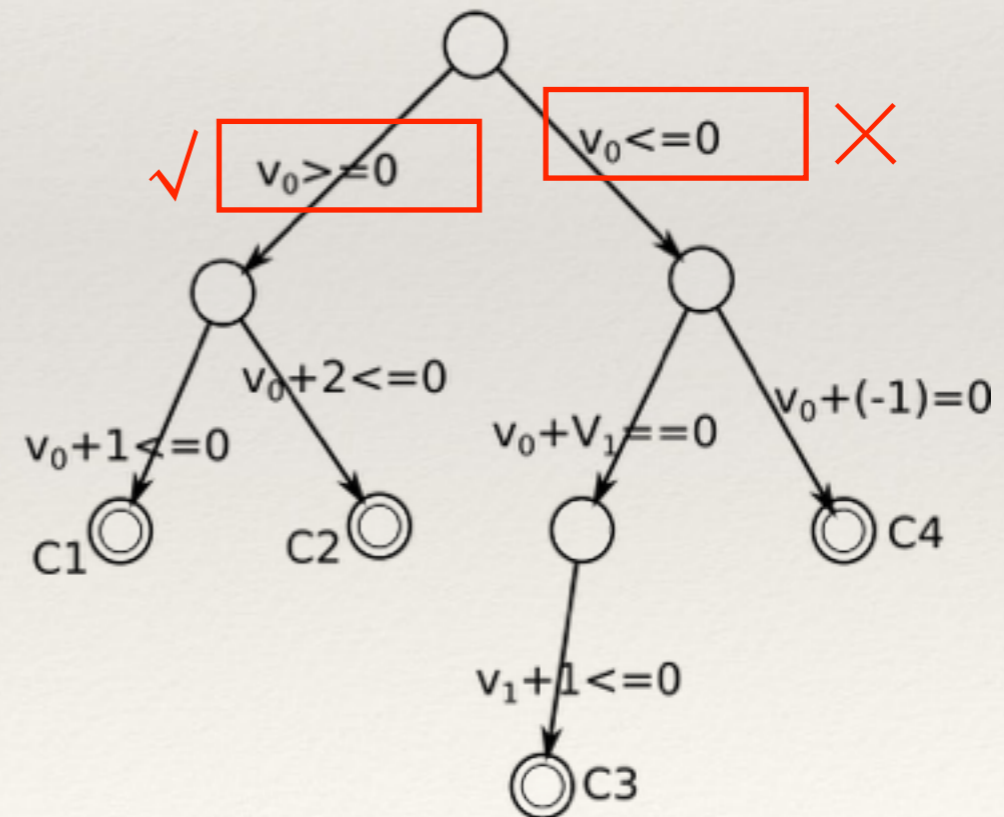
Target: $v_0 + (-1) \geq 0 \wedge v_0 + 3 \neq 0 \wedge v_0 + 4 \leq 0$

$IS_{\text{union}} = \{v_0 \geq 0, v_0 + 2 \leq 0, v_0 + 1 \leq 0\}$

Implication Partial Order Graph



Constraint Trie



Constraint Querying

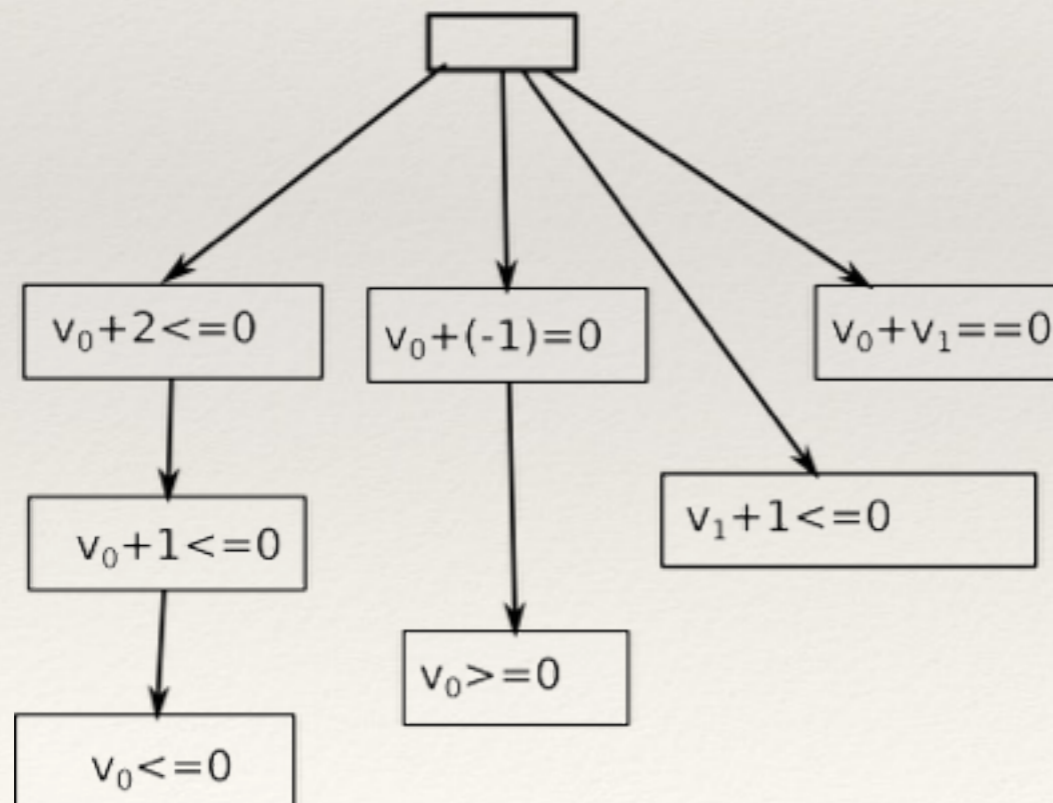
❖ Logical Subset Checking Algorithm

Target: $v_0 + (-1) \geq 0 \wedge v_0 + 3 \neq 0 \wedge v_0 + 4 \leq 0$

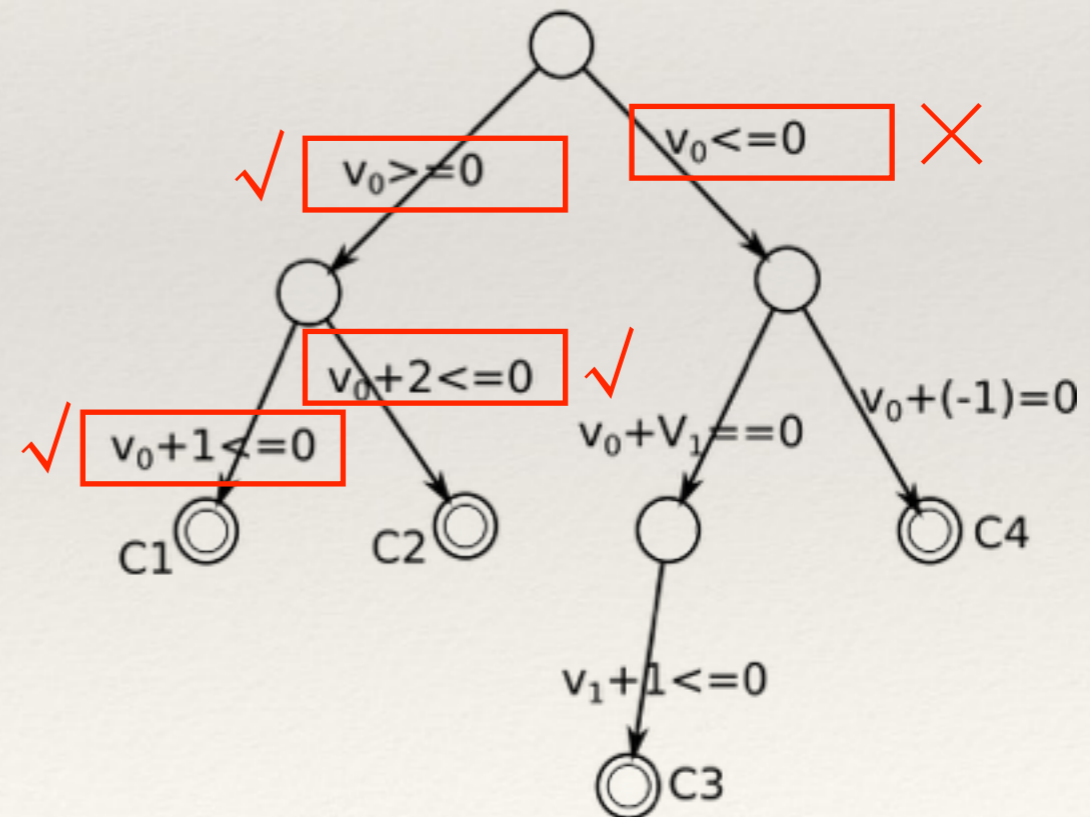
$IS_{\text{union}} = \{v_0 \geq 0, v_0 + 2 \leq 0, v_0 + 1 \leq 0\}$

We found two paths, so the target constraint is unsatisfiable.

Implication Partial Order Graph



Constraint Trie



Evaluation

- ❖ **Research Question**
 - ❖ Does GreenTrie achieve better reuse and save more time than other approaches (Green, KLEE) ?
- ❖ **Benchmarks**
 - ❖ 6 programs from Green (Willem Visser's FSE'12 paper)
 - ❖ 1 program from Guowei Yang's ISSTA 2012 paper.
- ❖ **Experiment scenarios**
 - ❖ (1) reuse in a single run of the program
 - ❖ (2) reuse across runs of different versions of the same program
 - ❖ (3) reuse across different programs

Evaluation

- ❖ Experiment setup
 - ❖ PC with a 2.5GHz Intel processor with 4 cores and 4Gb of memory
 - ❖ We implemented GreenTrie by extending Green
 - ❖ We implemented KLEE's subset/superset checking approach, and also integrated it into Green as an extension.
 - ❖ Symbolic executor: Symbolic Pathfinder (SPF)
 - ❖ Constraint Solver: Z3

Evaluation

❖ Reuse in a Single Run

Table 1: Experimental results of reuse in single run

Program	n_0	n_1	n_2	n_3	R'	R''	$t_0(\text{ms})$	$t_1(\text{ms})$	$t_2(\text{ms})$	$t_3(\text{ms})$	T'	T''
Trityp	32	28	28	28	0.00%	0.00%	1040	915	922	995	-8.74%	-7.92%
Euclid	642	552	464	464	15.94%	0.00%	5105	6503	7274	6311	2.95%	13.24%
TCAS	680	41	20	14	65.85%	30.00%	12742	3356	2182	2165	35.49%	0.78%
TreeMap1	24	24	24	24	0.00%	0.00%	871	942	947	882	6.37%	6.86%
TreeMap2	148	148	140	140	5.41%	0.00%	2918	2542	2851	2606	-2.52%	8.59%
TreeMap3	1080	956	833	806	15.69%	3.24%	21849	10729	11809	9871	8.00%	16.41%
BinTree1	84	41	25	25	39.02%	0.00%	1476	1103	1092	1027	6.89%	5.95%
BinTree2	472	238	133	118	50.42%	11.28%	4322	3648	3156	2872	21.27%	9.00%
BinTree3	3252	1654	939	873	47.22%	7.03%	36581	17197	14764	12041	29.98%	18.44%
BinomialHeap1	448	32	23	19	40.63%	17.39%	3637	2137	2046	2017	5.62%	1.42%
BinomialHeap2	3184	190	85	68	64.21%	20.00%	27165	7653	6442	6071	20.67%	5.76%
BinomialHeap3	23320	988	337	288	70.85%	14.54%	249224	28549	31892	21392	25.07%	32.92%
MerArbiter	60648	21	15	13	38.10%	13.33%	>10min	304726	290854	272813	10.47%	6.20%

n_i : the number of invocations to solver

t_i : running time for symbolic execution

$i=0$: SE without reuse

$i=1$: SE with Green

$i=2$: SE with KLEE's approach

$i=3$: SE with GreenTrie

Reuse improvement ratio: $R' = (n_1 - n_3) / n_1$ $R'' = (n_2 - n_3) / n_2$

Time improvement ratio: $T' = (t_1 - t_3) / t_1$ $T'' = (t_2 - t_3) / t_2$

Evaluation

❖ Reuse in a Single Run

Table 1: Experimental results of reuse in single run

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MerArbiter	60648	21	15	13	38.10%	13.33%	>10min	304726	290854	272813	10.47%	6.20%
total/average	94014	4913	3066	2880	41.38%	6.07%	/	390000	374012	341063	12.55%	9.35%

GreenTrie gets better reuse ratio and saves more time when the scale of execution increases.

Evaluation

❖ Reuse across Runs

Table 2: Experimental results of reuse across runs (program Euclid)

Changes	n_0	n_1	n_2	n_3	R'	R''	$t_1(\text{ms})$	$t_2(\text{ms})$	$t_3(\text{ms})$	T'	T''
ADD#1	492	432	5	3	99.54%	60.00%	3896	1375	1329	65.89%	3.35%
ADD#2	438	331	216	216	34.74%	0.00%	2830	3275	2284	19.29%	30.26%
ADD#3	220	170	32	2	98.82%	93.75%	1382	972	552	60.06%	43.21%
DEL#1	438	322	156	126	60.87%	19.23%	3428	2670	2171	36.67%	18.69%
DEL#2	492	426	350	134	68.54%	61.71%	3777	4483	2046	45.83%	54.36%
DEL#3	642	552	112	111	79.89%	0.89%	4649	2560	2049	55.93%	19.96%
MOD#1	642	552	464	463	16.12%	0.22%	4851	6899	4400	9.30%	36.22%
MOD#2	642	552	464	462	16.30%	0.43%	4765	7094	4351	8.69%	38.67%
MOD#3	642	551	442	433	21.42%	2.04%	4505	7481	4240	5.88%	43.32%
total/average	4648	3888	2241	1949	49.87%	13.03%	34083	36809	23422	31.28%	36.37%

Table 4: Experimental results of reuse across runs (program BinTree)

Changes	n_0	n_1	n_2	n_3	R'	R''	$t_1(\text{ms})$	$t_2(\text{ms})$	$t_3(\text{ms})$	T'	T''
ADD#1	5930	1689	803	746	55.83%	7.10%	17978	20355	11889	33.87%	41.59%
ADD#2	13358	3938	2618	2556	35.09%	2.37%	35382	105190	32465	8.24%	69.14%
ADD#3	15602	540	0	0	100.00%	0/0	18106	61586	17180	5.11%	72.10%
DEL#1	13358	3149	2216	2185	30.61%	1.40%	32134	126488	31002	3.52%	75.49%
DEL#2	5930	1154	599	0	100.00%	100.00%	13565	44789	10932	19.41%	75.59%
DEL#3	3252	1682	0	0	100.00%	0/0	12945	11482	4505	65.20%	60.76%
MOD#1	3252	1682	1080	1002	40.43%	7.22%	14553	16297	10628	26.97%	34.79%
MOD#2	3252	1680	716	632	62.38%	11.73%	14147	13784	7953	43.78%	42.30%
MOD#3	8310	2377	1068	964	59.44%	9.74%	22772	32889	14593	35.92%	55.63%
total/average	72244	17891	9100	8085	54.81%	11.15%	181582	432860	141147	22.27%	67.39%

GreenTrie gets better reuse ratio and saves more time than both Green and KLEE's approach.

Evaluation

❖ Reuse across Runs

Table 4: Experimental Results

Changes	n_0	n_1	n_2	n_3	R'	R''	t_1 (ms)	t_2 (ms)	t_3 (ms)	T'	T''
ADD#1	5930	1689	803	746	55.83%	7.10%	17978	20355	11889	33.87%	41.59%
ADD#2	13358	3938	2618	2556	35.09%	2.37%	35382	105190	32465	8.24%	69.14%
ADD#3	15602	540	0	0	100.00%	0/0	18106	61586	17180	5.11%	72.10%
DEL#1	13358	3149	2216	2185	30.61%	1.40%	32134	126488	31002	3.52%	75.49%
DEL#2	5930	1154	599	0	100.00%	100.00%	13565	44789	10932	19.41%	75.59%
DEL#3	3252	1682	0	0	100.00%	0/0	12945	11482	4505	65.20%	60.76%
MOD#1	3252	1682	1080	1002	40.43%	7.22%	14553	16297	10628	26.97%	34.79%
MOD#2	3252	1680	716	632	62.38%	11.73%	14147	13784	7953	43.78%	42.30%
MOD#3	8310	2377	1068	964	59.44%	9.74%	22772	32889	14593	35.92%	55.63%
total/average	72244	17891	9100	8085	54.81%	11.15%	181582	432860	141147	22.27%	67.39%

3421 constraints in store

Running time increases dramatically in KLEE's approach

GreenTrie gains better scalability than KLEE's approach

Evaluation

❖ Reuse across Programs

Numbers of reused constraints for Green, KLEE approach and GreenTrie

Table 5: Experimental results of reuse across programs

Program	Trityp	Euclid	TCAS	TreeMap	BinTree	BinomialHeap	MerArbiter
Trityp	/	0, 0, 3	0, 0, 3	4, 4	0, 2, 2	0, 6, 7	0, 0, 1
Euclid	0, 0, 1	/	2, 5, 5	0, 0, 0	0, 3, 4	0, 2, 2	0, 0, 2
TCAS	0, 0, 2	2, 2, 2	/	0, 0, 0	0, 2, 3	0, 3, 4	0, 3, 4
TreeMap	0, 0, 0	0, 0, 0	0, 0, 0	/	256, 326, 323	0, 0, 0	0, 0, 0
BinTree	0, 0, 0	0, 0, 0	0, 0, 0	256, 449, 470	/	0, 1, 1	0, 0, 0
BinomialHeap	2, 2, 5	2, 2, 5	2, 8, 6	0, 2, 3	1, 11, 10	/	0, 0, 0
MerArbiter	0, 1, 2	0, 2	0, 3	0, 0, 0	0, 0, 0	0, 0, 0	/

GreenTrie achieves more inter-programs reuse than Green.

In some cases, GreenTrie has a little less reuse than KLEE's approach.

The reason is that some constraints, which reuse the solution both across programs and in same program in GreenTrie, can only reuse constraints across programs in KLEE. Such constraints are counted for KLEE but not counted for GreenTrie

Conclusion and Future Work

❖ Contributions

- ❖ Logical subset/superset based reuse
- ❖ Trie-based store indexed with implication partial order graph
- ❖ Efficient logical subset/superset checking algorithms

❖ Future works

- ❖ Support more kinds of constraints other than linear integer constraints
- ❖ Reuse constraints which contains summaries
- ❖ Improve scalability for large-scale programs

Thanks

❖ Questions?