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Supplementary Material for "Boosting Local Search with Lagrangian Relaxation"

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Numerical Results for ALCMA$_{LANS}$

In this report, we present the complete experimental results of ALCMA$_{LANS}$, which is obtained by embedding Lagrangian relaxation Assisted Neighborhood Search (LANS) into the Accelerated Limit Crossing based Multilevel Algorithm (ALCMA). In the original version of ALCMA, interchange is applied as the underlying local search subroutine. To investigate the flexibility of LANS, we replace the local search within ALCMA, from interchange to LANS, and apply the variant over the benchmark instances.

The algorithms are implemented in C++, compiled with g++ 4.7 with flag -O3. The experiments are conducted on a Pentium IV 3.2 GHz PC with 4GB memory, running GNU/Linux with kernel 3.10. We employ the Euclidean (FL1400, PCB3038, and RL5934), the RW, and the GAP instances to conduct the experiment. We do not consider the other instances for the following reasons. For the ORLIB instances, applying LANS over random initial solutions could achieve the optimality. Besides, the RL11849 instances are not incorporated due to their scales.

For each benchmark instance, we independently execute ALCMA$_{LANS}$ for 9 times, which follows the experimental design of the existing work (Hansen and Mladenovic (1997); Resende and Werneck (2004); Ren et al (2012a)). Note that for the GAP instances, since not all the solutions are feasible (Kochetov et al (2005)), we shall repeat the executions of ALCMA$_{LANS}$, until 9 feasible output solutions are obtained.

The numerical results are presented in Tables 1–10. These tables are organized as follows. We present the instance-specific information in column 1. Column 2 lists the best known results in the literature, to the best of our knowledge (Resende and Werneck (2004); Pullan (2008); Ren et al (2012a,b, 2013)). Then, in columns 3–6, we give the minimum objective value achieved

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(min), average percentage error rate (%err), standard deviation (SD), and average running time (time).

Table 1 Numerical Results over FL1400 Instances

p	best	min	%err	SD	time
100	16551.20	16552.22	0.01	0.82	54.39
150	12026.41	12026.41	<0.01	0.02	165.23
200	9355.34	9355.34	<0.01	0.00	130.40
250	7737.72	7737.78	0.01	0.59	223.10
300	6611.60	6610.71	<0.01	0.88	177.30
350	5717.86	5720.05	0.08	1.48	277.59
400	5006.75	5006.75	<0.01	0.00	177.29
450	4468.28	4468.28	<0.01	0.01	186.90
500	4044.10	4046.16	0.06	0.65	387.18

Table 2 Numerical Results over PCB3038 Instances

p	best	min	%err	SD	time
100	352609.60	352609.86	<0.01	4.93	709.00
150	281163.12	281163.12	<0.01	14.11	644.97
200	238344.20	238344.20	<0.01	0.00	500.13
250	209206.90	209206.90	<0.01	5.12	362.47
300	187689.40	187686.24	<0.01	5.09	230.75
350	170919.46	170919.46	<0.01	0.20	182.53
400	157027.21	157027.21	<0.01	0.89	164.61
450	145362.91	145362.91	<0.01	0.28	158.49
500	135447.39	135447.39	<0.01	1.88	186.46
550	126825.24	126825.24	<0.01	1.68	142.21
600	119059.77	119058.09	<0.01	0.91	144.49
650	112017.65	112017.65	<0.01	0.68	182.32
700	105823.96	105822.49	<0.01	0.78	228.58
750	100331.25	100329.88	0.01	5.83	314.79
800	95374.17	95372.46	<0.01	2.17	304.05
850	90983.64	90989.41	0.01	2.38	396.56
900	86972.78	86972.15	<0.01	2.88	445.38
950	83267.38	83264.78	<0.01	2.70	518.49
1000	79842.01	79842.01	<0.01	1.79	563.18

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Table 3 Numerical Results over RL5934 Instances

p	best	min	$\%err$	SD	time
100	2724740.61	2724740.61	<0.01	75.00	6286.95
150	2147756.39	2147756.39	<0.01	0.00	5106.34
200	1807411.06	1807411.06	<0.01	24.03	3802.35
250	1569712.98	1569712.98	<0.01	4.04	2320.40
300	1393951.74	1393951.74	<0.01	3.18	1395.12
350	1256599.00	1256599.00	<0.01	0.00	1981.84
400	1145271.28	1145271.28	<0.01	32.39	1903.98
450	1053024.59	1053024.59	<0.01	5.86	1912.07
500	973920.22	973920.22	<0.01	47.61	1161.51
600	848217.37	848217.37	<0.01	1.25	1438.31
700	751972.31	751970.11	<0.01	0.26	1728.10
800	676724.24	676724.24	<0.01	0.30	1669.73
900	613308.88	613308.88	<0.01	0.37	1739.76
1000	558783.56	558783.56	<0.01	3.52	2257.81
1100	511778.61	511764.80	<0.01	4.61	2017.19
1200	470264.28	470264.28	<0.01	0.46	2083.42
1300	433543.91	433543.91	<0.01	0.00	2344.65
1400	401820.03	401820.03	<0.01	0.00	2558.66
1500	373987.02	373987.02	<0.01	0.00	2070.97

Table 4 Numerical Results over RW100 Instances

p	best	min	$\%err$	SD	time
10	530.00	530.00	<0.01	0.00	5.89
20	277.00	277.00	<0.01	0.00	3.09
30	213.00	213.00	<0.01	0.00	1.13
40	187.00	187.00	<0.01	0.00	1.16
50	172.00	172.00	<0.01	0.00	0.66

Table 5 Numerical Results over RW250 Instances

p	best	min	$\%err$	SD	time
10	3691.00	3691.00	0.27	14.90	64.11
25	1360.00	1364.00	0.31	0.67	26.99
50	713.00	713.00	<0.01	0.00	10.61
75	523.00	523.00	<0.01	0.00	2.79
100	444.00	444.00	<0.01	0.00	2.32
125	411.00	411.00	<0.01	0.00	2.05

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Table 6 Numerical Results over RW500 Instances

p	best	min	$\%err$	SD	time
10	16108.00	16108.00	0.70	103.45	595.31
25	5626.00	5681.00	1.89	45.59	249.48
50	2626.00	2626.00	0.47	7.92	73.39
75	1757.00	1757.00	<0.01	0.00	43.95
100	1379.00	1379.00	0.27	3.97	36.56
150	1024.00	1024.00	<0.01	0.00	6.38
200	893.00	893.00	<0.01	0.00	7.60
250	833.00	833.00	<0.01	0.00	6.80

Table 7 Numerical Results over RW1000 Instances

p	best	min	$\%err$	SD	time
10	67811.00	67811.00	0.37	143.31	7374.14
25	24896.00	24979.00	1.38	154.20	3718.94
50	11259.00	11344.00	1.43	71.59	953.60
75	7134.00	7176.00	1.00	30.93	373.82
100	5210.00	5235.00	1.11	29.42	221.28
200	2704.00	2705.00	0.16	3.71	141.04
300	2018.00	2018.00	0.01	0.33	43.14
400	1734.00	1734.00	<0.01	0.00	33.60
500	1614.00	1614.00	<0.01	0.00	28.99

Table 8 Numerical Results over GAP-A Instances

instance	best	min	%err	SD	time
1032PM	127.00	127.00	1.05	3.04	3.22
1132PM	163.00	163.00	5.52	8.62	0.94
1232PM	123.00	123.00	6.68	9.60	2.43
1332PM	164.00	164.00	0.27	1.01	1.35
1432PM	150.00	150.00	4.15	5.47	1.64
1532PM	158.00	158.00	<0.01	0.00	1.25
1632PM	141.00	141.00	1.58	2.64	1.39
1732PM	157.00	157.00	0.42	1.00	1.62
1832PM	135.00	135.00	4.53	3.44	1.30
1932PM	146.00	146.00	0.53	0.44	0.91
2032PM	150.00	150.00	6.15	6.74	1.04
2132PM	140.00	140.00	6.35	9.71	1.50
2232PM	145.00	145.00	<0.01	0.00	1.93
2332PM	172.00	172.00	1.61	2.64	1.06
2432PM	137.00	138.00	3.97	4.25	2.15
2532PM	153.00	153.00	1.23	5.67	0.98
2632PM	164.00	164.00	3.32	3.94	1.11
2732PM	123.00	123.00	10.30	11.34	2.53
2832PM	145.00	145.00	4.90	4.01	2.64
2932PM	155.00	155.00	6.09	8.96	1.44
3032PM	113.00	113.00	2.36	2.60	3.45
3132PM	130.00	130.00	3.68	8.24	4.72
3232PM	157.00	172.00	9.55	0.00	0.93
332PM	154.00	154.00	4.76	6.84	2.61
432PM	155.00	155.00	<0.01	0.00	0.89
532PM	150.00	151.00	2.30	1.59	1.64
632PM	162.00	162.00	3.84	5.09	1.28
732PM	157.00	157.00	<0.01	0.00	1.41
832PM	136.00	136.00	1.80	4.85	1.73
932PM	133.00	133.00	1.42	5.67	2.22

Table 9 Numerical Results over GAP-B Instances

instance	best	min	$\%err$	SD	time
1031PM	165.00	165.00	<0.01	0.00	0.96
1131PM	162.00	162.00	<0.01	0.00	0.97
1231PM	136.00	142.00	9.80	5.24	1.04
1331PM	135.00	138.00	5.84	6.23	1.74
1431PM	129.00	131.00	6.20	7.35	2.32
1531PM	130.00	130.00	6.58	5.50	3.33
1631PM	140.00	140.00	5.63	5.90	2.37
1731PM	174.00	174.00	5.36	11.07	0.93
1831PM	134.00	134.00	3.07	6.31	1.70
1931PM	136.00	136.00	3.92	8.63	2.06
2031PM	131.00	131.00	3.31	7.84	1.79
2131PM	126.00	126.00	4.85	8.78	1.88
2231PM	179.00	179.00	<0.01	0.00	0.93
2331PM	134.00	134.00	9.12	10.00	1.35
2431PM	137.00	137.00	4.70	7.88	1.75
2531PM	124.00	124.00	5.02	6.82	2.50
2631PM	131.00	131.00	8.91	7.97	2.02
2731PM	159.00	159.00	<0.01	0.00	0.96
2831PM	151.00	151.00	<0.01	0.00	1.40
2931PM	132.00	137.00	9.09	5.41	1.40
3031PM	144.00	144.00	5.48	10.93	1.38
3131PM	158.00	158.00	3.02	9.56	1.34
3231PM	125.00	125.00	12.62	12.59	0.96
331PM	123.00	136.00	15.18	5.00	2.73
431PM	132.00	132.00	9.85	10.27	1.27
531PM	135.00	135.00	7.82	12.71	1.64
631PM	140.00	140.00	7.46	11.16	0.92
731PM	130.00	133.00	12.39	9.51	1.71
831PM	138.00	138.00	4.83	7.31	2.05
931PM	172.00	172.00	<0.01	0.00	0.94

Table 10 Numerical Results over GAP-C Instances

instance	best	min	$\%err$	SD	time
1033PM	138.00	138.00	9.74	14.61	0.93
1133PM	147.00	148.00	8.24	10.49	0.96
1233PM	142.00	142.00	3.60	5.16	1.78
1333PM	140.00	140.00	6.51	12.82	0.96
1433PM	152.00	153.00	7.24	8.12	0.97
1533PM	133.00	133.00	15.20	12.82	0.97
1633PM	141.00	144.00	4.81	4.55	1.71
1733PM	134.00	134.00	9.20	9.01	1.01
1833PM	139.00	144.00	8.07	7.63	1.40
1933PM	137.00	137.00	9.49	8.28	1.39
2033PM	140.00	140.00	13.02	12.11	1.01
2133PM	138.00	146.00	12.64	6.86	0.96
2233PM	121.00	121.00	7.35	11.83	1.35
2333PM	133.00	139.00	11.19	4.01	1.37
2433PM	139.00	141.00	10.31	12.92	0.99
2533PM	131.00	144.00	17.39	7.98	1.84
2633PM	132.00	132.00	11.45	11.71	1.83
2733PM	139.00	153.00	18.31	9.40	0.98
2833PM	137.00	137.00	10.06	8.06	1.41
2933PM	124.00	124.00	3.49	5.57	0.97
3033PM	137.00	148.00	12.17	6.20	0.93
3133PM	141.00	141.00	10.32	7.11	1.00
3233PM	129.00	129.00	13.26	9.98	0.96
333PM	147.00	153.00	10.51	8.34	1.04
433PM	145.00	145.00	5.67	5.83	1.08
533PM	142.00	142.00	5.32	8.25	1.48
633PM	144.00	144.00	10.11	11.64	1.05
733PM	137.00	141.00	12.98	7.93	1.49
833PM	144.00	147.00	12.50	11.17	2.02
933PM	130.00	130.00	5.38	11.06	1.84